

# Recommender Systems

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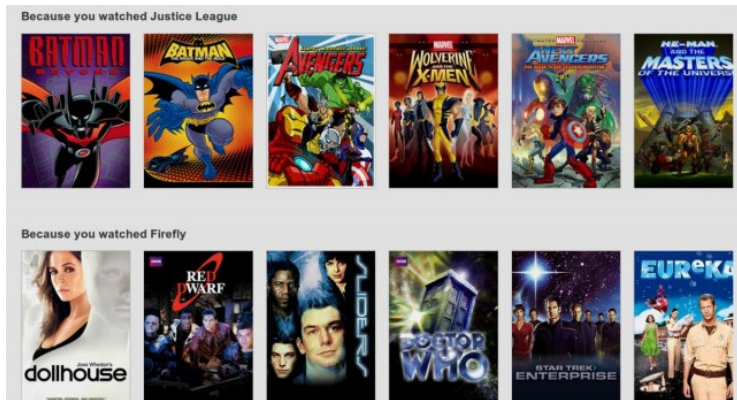
# Tools so far

We already know many things that we can use to recommend new content.

- ▶ predict rating of a restaurant
- ▶ predict if a user is going to respond to a marketing campaign
- ▶ predict if user is going to like a music

# Why do we need recommendations?

Help users discover new content



# Why do we need recommendations?

Help users find what they have been already looking for.

Prominently place content as a part of the user interface.

The screenshot shows a video player interface for the movie "Harry Potter and the Sorcerer's Stone" (2001). The title is displayed at the top left, followed by the year "2001", a "PG-13" rating, and a "CC" icon. Below the title is a large image of Harry Potter. To the right of the image are a star rating of 4.5 (based on 2,829 reviews) and a "IMDb 7.5/10" badge. A "Watch Trailer" button is located below the image. The main description reads: "Based on the wildly popular J.K. Rowling's book about a young boy who on his eleventh birthday discovers he is the orphaned boy of two powerful wizards and has unique magical powers." Below this, it lists the starring cast: "Richard Harris, Maggie Smith" and the runtime: "2 hours, 33 minutes". It also states "Available to watch on supported devices." On the right side of the player, there are two orange buttons: "Rent HD \$3.99" and "Buy HD \$13.99". Below these are links for "Redeem a gift card" and "Add to V". At the bottom left, there is a "Share" button with icons for email, Facebook, and Twitter. At the bottom right, there is a small text line: "By placing your order, you agree to our [Terms of Use](#). Sold by Amazon Digital Services, Inc."

## Customers Who Watched This Item Also Watched



# Why do we need recommendations?

Help users find complementary products



## Levi's Men's 501 Original Shrink To Fit Jean

★★★★☆ • 1,353 customer reviews | 6 answered questions

Price: \$17.33 - \$59.99 & FREE Returns on some sizes and colors. [Details](#)

Sale: Lower price available on select options

Fit: As expected (71%)

Size:

Select [Size Chart](#)

Color: Rigid STF



- 100% Cotton
- Imported
- Machine Wash
- Straight-leg jean in shrink-to-fit denim that fades with repeat washings
- Five-pocket styling
- Button Fly
- 17.25-leg opening
- Actual coloration may vary from garment to garment due to specific wash processes

## Customers Who Bought This Item Also Bought



Levi's Men's 40MM Reversible Belt With Gunmetal Buckle  
★★★★☆: 555  
\$13.99 - \$19.99



Carhartt Men's Twill Long Sleeve Work Shirt Button Front  
★★★★☆: 132  
\$24.99 - \$25.99



Wrangler Men's Sport Western Snap Long Sleeve Shirt  
★★★★☆: 147  
\$19.95 - \$26.50



Burnside Men's Locked Long-Sleeve Woven Shirt  
★★★★☆: 76  
\$15.01 - \$42.00



Levi's Men's 514 Straight-Fit Corduroy Pant  
★★★★☆: 30  
\$42.99



U.S. Polo Assn. Men's Slim Fit Solid Pique Polo Shirt  
★★★★☆: 71  
\$11.82 - \$30.99



Clarks Originals Men's Desert Boot  
★★★★☆: 2,643  
**#1 Best Seller** in Men's Chukka Boots  
\$56.86 - \$199.90

# Why do we need recommendations?

Help users find substitute products



## Levi's Men's 501 Original Shrink To Fit Jean

★★★★☆ • 1,353 customer reviews | 6 answered questions

Price: \$17.33 - \$59.99 & FREE Returns on some sizes and colors. [Details](#)

Sale: Lower price available on select options.

Fit: As expected (71%)

Size:

Select Size Chart

Color: Rigid STF



- 100% Cotton
- Imported
- Machine Wash
- Straight-leg jean in shrink-to-fit denim that fades with repeat washings
- Five-pocket styling
- Button Fly
- 17.25-leg opening
- Actual coloration may vary from garment to garment due to specific wash processes

## Customers Who Viewed This Item Also Viewed



Levi's Men's Jeans 501  
Original Fit  
★★★★☆ 4,855  
\$34.99 - \$64.99



Levi's Men's 505 Regular  
Fit Jean  
★★★★☆ 5,479  
**#1 Best Seller** in Men's  
Jeans  
\$39.49 - \$68.00



Levi's Men's 514 Straight  
Jean  
★★★★☆ 3,089  
\$19.15 - \$58.00



Levi's Men's Big-Tall 501  
Shrink To Fit Jean  
★★★★☆ 263  
\$39.99 - \$59.99



Levi's Men's 501 Colored  
Rigid Shrink-to-Fit Jean  
(Clearance) Light Gray  
★★★★☆ 63  
\$16.70 - \$49.99



Levi's Men's 501 Colored  
Rigid Shrink-to-Fit Jean  
(Clearance) Blue Green  
Rigid  
★★★★☆ 80  
\$16.83 - \$49.99



Levi's Men's Big & Tall 501  
Original-Fit Jean  
★★★★☆ 241  
\$46.99 - \$59.99

# Why do we need recommendations?



















Personalize user experience based on the feedback

Recommend products based on our interests

The image shows a video player interface. At the top, there is a dark blue control bar with icons for thumbs down, thumbs up, pause, play, and volume. Below the icons, a progress bar shows the video is at 0:09 out of a total duration of -2:14. Below the control bar, there is a section labeled 'My Profile' with a person icon. This section contains two video thumbnails: the first is titled 'GAELIC STORM What's the Rumpus?' and the second is 'Celtic Woman A NEW JOURNEY DELUXE EDITION'. To the right of these thumbnails is an advertisement for 'Dove MEN+CARE BODY WASH'. The ad features a bottle of the product and a 'BUY NOW' button. A small 'AdChoices' icon is visible in the top right corner of the ad area.

# Why do we need recommendations?

Who to follow?

	<b>Joe Blitzstein</b> @stat110 Statistics professor at Harvard; statistician and data scientist; probability and paradoxes; Bayesian frequentist reconciliation; chess. Followed by Bert Huang, Sherri Rose and Hal Daumé III.		
	<b>Sean J. Taylor</b> @seanjtaylor Social Scientist. Hacker. Facebook Data Science Team. Keywords: Experiments, Causal Inference, Statistics, Machine Learning, Economics. Followed by José Pablo González, Sherri Rose and John D. Cook.		
	<b>Daniel Roy</b> @roydanroy Asst Professor Of Stats at UofT working also in theoretical computer science, machine learning, and probability. I like randomness. Followed by Arthur Gretton, Dino Sejdinovic and Neil Lawrence.		
	<b>Jason Baldridge</b> @jasonbaldridge Co-founder of @PeoplePattern and Associate Professor of Computational Linguistics, UTAustin. Followed by José Pablo González, Jonathan Clark and petrcek.		
	<b>Nando de Freitas</b> @NandoDF Oxford University Prof & Google DeepMind Scientist		
	<b>Ryan Adams</b> @ryan_p_adams Computer Science Professor, Machine Learning Researcher @Twitter, Entrepreneur, Podcaster, Dad, Sports Fan Followed by Bert Huang, Hal Daumé III and Tomasz Malisiewicz.		



# Why do we need recommendations?

Help us find things we like

Results for *'the guild'*



**Play + All**

☆☆☆☆

**The Guild: Seasons 1 & 2**  
2007 **NR** 2 discs / 2 episodes

Actress and former "World of Warcraft" addict Felicia Day created and stars in this Web series about a quirky group of gamers who take part in all sorts of heroics online but are far less skilled at navigating real life.

**Starring:** Felicia Day, Jeff Lewis  
**Creator:** Felicia Day  
**Genre:** TV Sitcoms  
**Format:** DVD and streaming (HD available)

☆☆☆☆ **3.5** Our best guess for Mike

# Why do we need recommendations?

- ▶ Discover new content
- ▶ Help us find what we are looking for
- ▶ Personalize content based on our feedback/interests
- ▶ Help us find things we like
- ▶ ...

All of the above problems are slightly different from each other.

However, at the base of all of them is the need to model preferences, opinions and behaviour of users.

# Popularity

What are people viewing now

## Trends · [Change](#)

### #NationalStressAwarenessDay

77.7K Tweets about this trend

### #YouHadMeAt

26.5K Tweets about this trend

### #FoxLake











Just started trending

### #BAYvAFC

42.7K Tweets about this trend

### San Diego

40.3K Tweets about this trend

	MOST EMAILED	MOST VIEWED
1.	Missing Italian Marathoner Found on New York Subway, Still in His Running Gear	
2.	Beyond the Honeycrisp Apple	
3.	ESSAY On the Longest Hiking Trails, a Woman Finds Equal Footing	
4.	THE UPSHOT Stressed, Tired, Rushed: A Portrait of the Modern Family	
5.	ABOUT NEW YORK Sidelight to a Spy Saga: How a Brooklyn Newsboy's Nickel Would Turn Into a Fortune	
6.	DAVID BROOKS The Evolution of Simplicity	
7.	FRANK BRUNI The Catholic Church's Sins Are Ours	
8.	Robin Williams's Widow Points to Dementia as a Suicide Cause	
9.	OP-ED   THOMAS B. EDGALL Why Are Asian-Americans Such Loyal Democrats?	
10.	Review: Resettling the Meaning of Home in 'Brooklyn,' With Saoirse Ronan	

[Go to Complete List »](#)

## Limitations

- ▶ no context (what is my intention now)
- ▶ no personalization

# Predicting rating using existing models

Build a model using features of a user and an item

**Trollhunter (English Subtitled)** 2011 PG-13 CC



★★★★★ 665 IMDb 7/10

A trio of college students documents every move of a grizzled, unlikely hero - the trollhunter - risking their lives to uncover the secrets of creatures only thought to exist in fairy tales.

**Starring:** Otto Jespersen, Glenn Erland Tosterud  
**Runtime:** 1 hour, 44 minutes  
Available to watch on [supported devices](#).

David L. Alberta

Grand Rapids, MI

Reviewer ranking: #1,450,021

**89% helpful**

votes received on reviews  
(163 of 184)

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**PUBLIC ACTIVITY**

[Reviews \(15\)](#)

**OTHER**

[Give Profile Feedback to Amazon](#)

Reviewed



TrollHunter

120 of 128 people found this review helpful

★★★★★ **Quirky, intelligent and just plain fun to watch**

June 26, 2011

This film demonstrates how mockumentaries should be made. The film never takes itself too serious and is just plain fun. The trolls look like doofy monsters from the Grimms Fairy-Tales and are not like the monsters you see in animated kid flicks. As Troll Hunter explains, fairy-tales and real life are not the same. Trolls are just dumb animals and as animals they act on their instinctive behavior. And why are the trolls in the movie being hunted down? Well, you will see if you watch the movie. [...Read more](#)

## Product Details

Genres	Fantasy, Drama, International, Comedy, Horror
Director	André Øvredal
Starring	Otto Jespersen, Glenn Erland Tosterud
Supporting actors	Johanna Marck, Tomas Alf Larsen, Urmila Berg-Domaas, Hans Morten Hansen, Robert Stoltenberg, Knut Nærum, Eirik Bech, Inge Erik Høyenstad, Tom Jørgensen, Benedicte Aubert Ringnes, Magne Skjævesland, Torunn Ledemel Stokkeland, Finn Norvald Øvredal, Kaja Halden Aarrestad, Robin De Lano, Jens Stoltenberg
Studio	Magnolia
MPAA rating	PG-13 (Parental Guidance Suggested)
Captions and subtitles	English <a href="#">Details</a>
Rental rights	48 hour viewing period. <a href="#">Details</a>
Purchase rights	Stream instantly and download to 2 locations <a href="#">Details</a>
Format	Amazon Video (streaming online video and digital download)

# Predicting rating using existing models

## The Music Genome Project

<https://www.pandora.com/about/mgp>

... Each song in the Music Genome Project is analyzed using up to 450 distinct musical characteristics by a trained music analyst. These attributes capture not only the musical identity of a song, but also the many significant qualities that are relevant to understanding the musical preferences of listeners. The typical music analyst working on the Music Genome Project has a four-year degree in music theory, composition or performance, has passed through a selective screening process and has completed intensive training in the Music Genome's rigorous and precise methodology. ...

# Predicting rating using existing models




**Lady Gaga**

**PANDORA®**

Just Dance (Remix Single)

Just Dance (Redone Remix F. Kardinal Offishall)

 Play Sample

 **Create A Station**

**Bookmark This Track**

**Buy on iTunes**

**Buy CD From Amazon**

**Buy From Amazon MP3**

## Features Of This Track

electronica roots  
trip hop roots  
r&b influences  
funk influences  
beats made for dancing  
unsyncopated ensemble rhythms  
straight drum beats  
a female vocal  
clear pronunciation  
a rhythmic intro  
use of modal harmonies  
the use of chordal patterning  
melodic part writing  
use of strings  
subtle use of arpeggiated synths  
affected synths

Obtaining features may be expensive

# Predicting rating using existing models

Build a model using features of a user and an item

- ▶ linear model
- ▶ decision tree
- ▶ boosting model
- ▶ ...

$$\text{rating} = f(\text{user features}, \text{item features})$$

# Predicting rating using existing models

We could predict how many stars a user would give.

We could predict whether a user would give a high rating if they purchase an item.

We could predict whether a user will like an item.

Based on the predicted rating, the system would recommend

- ▶ diverse set of items
- ▶ new/unseen content
- ▶ something that the user likes or searches for



# Predicting rating using existing models

We train a model based on user's past feedback and various features that characterize users and items.

Recommender system provides a rating and we provide recommendations based on this.

Similar to the example in marketing where customers are targeted based on  $\hat{p}$ .

# Predicting rating using existing models

Approach does not suffer from a cold-start problem

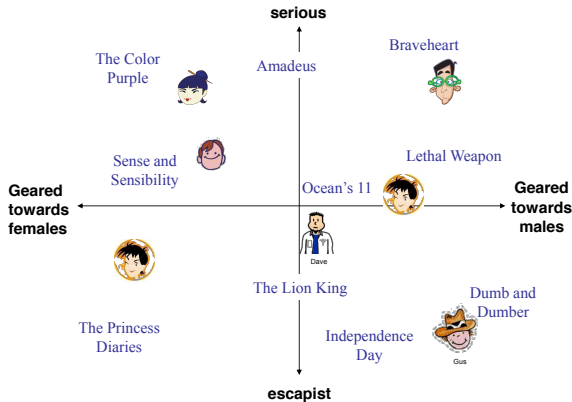
- ▶ Rate new movie from features of other movies user liked

Limitations

- ▶ sometimes, we do not have access to features
- ▶ often does not perform as well as collaborative filtering methods

# Learning relationships

Recommender systems can uncover/model relationships between users and items they are evaluating, using historical ratings.



# Collaborative filtering

Data we may have (example from Netflix challenge)

Training data

user	movie	date	score
1	21	5/7/02	1
1	213	8/2/04	5
2	345	3/6/01	4
2	123	5/1/05	4
2	768	7/15/02	3
3	76	1/22/01	5
4	45	8/3/00	4
5	568	9/10/05	1
5	342	3/5/03	2
5	234	12/28/00	2
6	76	8/11/02	5
6	56	6/15/03	4

Test data

user	movie	date	score
1	62	1/6/05	?
1	96	9/13/04	?
2	7	8/18/05	?
2	3	11/22/05	?
3	47	6/13/02	?
3	15	8/12/01	?
4	41	9/1/00	?
4	28	8/27/05	?
5	93	4/4/05	?
5	74	7/16/03	?
6	69	2/14/04	?
6	83	10/3/03	?

# Collaborative filtering

Data commonly represented as a rating matrix.

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
items	1	1		3			5			5		4	
	2			5	4			4			2	1	3
	3	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	6	1		3		3			2			4	



- unknown rating



- rating between 1 to 5

# Collaborative filtering

- ▶ a user viewed/bought an item (1)
- ▶ a user did not view/buy an item (0)

	users										
movies	1	3			5			5	4		
			5	4			4		2	1	3
	2	4		1	2		3	4	3	5	
		2	4		5			4		2	
			4	3	4	2				2	5
	1		3		3			2			4



	users											
movies	1	0	1	0	0	1	0	0	1	0	1	0
	0	0	1	1	0	0	1	0	0	1	1	1
	1	1	0	1	1	0	1	0	1	1	1	0
	0	1	1	0	1	0	0	1	0	0	1	0
	0	0	1	1	1	1	0	0	0	0	1	1
	1	0	1	0	1	0	0	1	0	0	1	0
	1	0	1	0	1	0	0	1	0	0	1	0

# Collaborative filtering

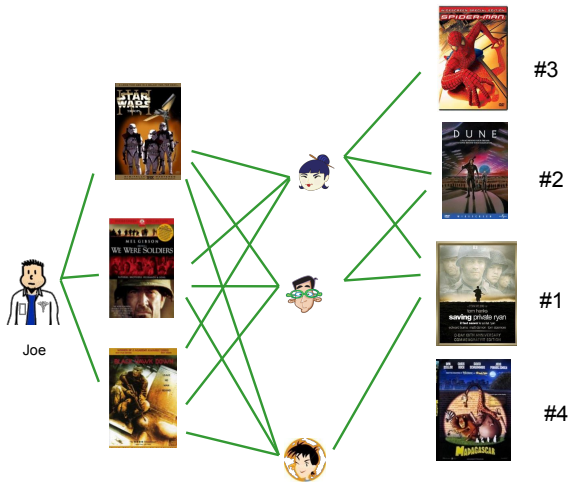
- ▶ 1 user viewed/bought an item and liked it
- ▶ -1 user viewed/bought an item and did not like it
- ▶ 0 user did not view/buy an item

	users											
movies	1		3			5			5		4	
			5	4			4			2	1	3
	2	4		1	2		3		4	3	5	
		2	4		5			4			2	
			4	3	4	2					2	5
	1		3		3			2				4



	users											
movies	-1	0	-1	0	0	1	0	0	1	0	1	0
	0	0	1	1	0	0	1	0	0	-1	-1	-1
	-1	1	0	-1	-1	0	-1	0	1	-1	1	0
	0	-1	1	0	1	0	0	1	0	0	-1	0
	0	0	1	-1	1	-1	0	0	0	0	-1	1
	-1	0	-1	0	-1	0	0	-1	0	0	1	0

# Basic idea





# Basic idea

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
items	1	1		3		??	5			5		4	
	2			5	4			4			2	1	3
	3	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	6	1		3		3			2			4	



- unknown rating



- rating between 1 to 5

# Basic idea

	users											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		??	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	



- unknown rating



- rating between 1 to 5

similarity

$$s_{13} = 0.2$$

$$s_{16} = 0.3$$

Predict rating using  
weighted average

$$\frac{0.2 \cdot 2 + 0.3 \cdot 3}{0.2 + 0.3} = 2.6$$

# Basic idea

(user, user) similarity to recommend items

(item, item) similarity to recommend new items that were also liked by the same users

The oldest known collaborative filtering method.

See Amazon recommendation system at scale:

<http://www.cs.umd.edu/~samir/498/Amazon-Recommendations.pdf>

# Similarities

How do we measure (user, user) similarity or (item, item) similarity?

- ▶ Euclidean distance
- ▶ Jaccard similarity
- ▶ Cosine similarity
- ▶ Pearson correlation

Notation:

$$\text{rating}(\text{user}, \text{item}) = r(u, i) = r_{ui}$$

$I_u$  = set of items purchased by user  $u$

$U_i$  = set of users who purchased by item  $i$

## Euclidean distance

Distance between item  $i_1$  and item  $i_2$  is

$$\text{dist}^2(i_1, i_2) = \sum_u (r_{u,i_1} - r_{u,i_2})^2$$

If each rating is 0 or 1 (user bought item or did not), then the distance becomes

$$\begin{aligned} \text{dist}^2(i_1, i_2) = & \#\{\text{users that bought } i_1, \text{ but not } i_2\} \\ & + \#\{\text{users that bought } i_2, \text{ but not } i_1\} \end{aligned}$$

# Euclidean distance

Example:

$$U_1 = \{1, 4, 8, 9, 11, 23, 25, 34\}$$

$$U_2 = \{1, 4, 6, 8, 9, 11, 23, 25, 34, 35, 38\}$$

$$U_3 = \{4\}$$

$$U_4 = \{5\}$$

What is the distance between items 1 and 2?

What is the distance between items 3 and 4?

# Euclidean distance

What is the distance between items 1 and 2?  $\rightarrow 3$

What is the distance between items 3 and 4?  $\rightarrow 2$

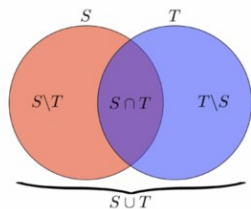
Problem:

- ▶ Favors small sets, even if they have few elements in common.

# Jaccard similarity

Measures similarity between sets

$$\begin{aligned} \text{Jaccard}(U_i, U_j) &= \frac{|U_i \cap U_j|}{|U_i \cup U_j|} \\ &= \frac{\text{bought } i \text{ and } j}{\text{bought } i \text{ or } j} \end{aligned}$$



Key idea: normalize by popularity



# Jaccard similarity

- ▶ Maximum of 1 if two items were purchased by the same set of users or if the two users purchased exactly the same set of items.
- ▶ Minimum of 0 if the two items were purchased by completely disjoint sets of users or if the two users purchased completely disjoint sets of items.

# Jaccard similarity in action

How does amazon generate their recommendations?

<http://www.cs.umd.edu/~samir/498/Amazon-Recommendations.pdf>

A user is looking at



$U_i$  is the set of users who viewed this product.

Rank products according to  $\frac{|U_i \cap U_j|}{|U_i \cup U_j|}$



.86



.84



.82



.79



...



## Cosine similarity

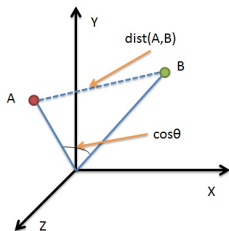
Jaccard similarity works only on 0/1 data

Cosine similarity works on arbitrary data.

	users											
movies	-1	0	-1	0	0	1	0	0	1	0	1	0
	0	0	1	1	0	0	1	0	0	-1	-1	-1
	-1	1	0	-1	-1	0	-1	0	1	-1	1	0
	0	-1	1	0	1	0	0	1	0	0	-1	0
	0	0	1	-1	1	-1	0	0	0	0	-1	1
	-1	0	-1	0	-1	0	0	-1	0	0	1	0

- ▶ 1 user viewed/bought an item and liked it
- ▶ -1 user viewed/bought an item and did not like it
- ▶ 0 user did not view/buy an item

# Cosine similarity

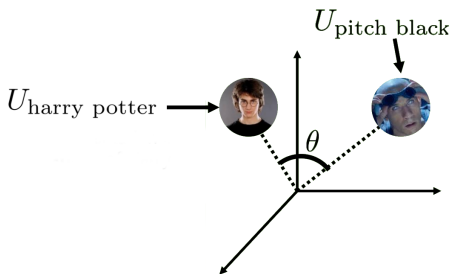


$$\text{similarity}(A, B) = \cos(\theta) = \frac{A \cdot B}{\|A\| \cdot \|B\|}$$

- ▶  $\cos(\theta) = 1$  ( $\theta = 0$ ) A and B point in the same direction
- ▶  $\cos(\theta) = -1$  ( $\theta = 180$ ) A and B point in the opposite direction
- ▶  $\cos(\theta) = 0$  ( $\theta = 90$ ) A and B are orthogonal

## Cosine similarity

Each item is represented by a vector of users' ratings



$$U_{\text{harry potter}} = (0, 1, 1) \quad U_{\text{pitch black}} = (1, 1, 0)$$

$$\text{similarity} = \frac{(0, 1, 1) \cdot (1, 1, 0)}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

# Pearson correlation

Very similar to cosine similarity

Cosine similarity would fail if naively applied to ratings

Example:

$$\begin{aligned} R_1 &= (1, 1, 1) \\ R_2 &= (5, 5, 5) \end{aligned} \longrightarrow \text{similarity} = 1$$

Pearson correlation will solve this by removing average from the rating vector

# Pearson correlation

User ratings for item i:

1	?	?	5	5	3	?	?	?	4	2	?	?	?	?	4	?	5	4	1	?
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

User ratings for item j:

?	?	4	2	5	?	?	1	2	5	?	?	2	?	?	3	?	?	?	5	4
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Pearson correlation computed over shared support

$$s_{ij} = \frac{\sum_{u \in U_i \cap U_j} (r_{ui} - \bar{r}_i)(r_{uj} - \bar{r}_j)}{\sqrt{\sum_{u \in U_i \cap U_j} (r_{ui} - \bar{r}_i)^2 \cdot \sum_{u \in U_i \cap U_j} (r_{uj} - \bar{r}_j)^2}}$$

# Pearson correlation vs cosine similarity

Pearson correlation

$$s_{ij} = \frac{\sum_{u \in U_i \cap U_j} (r_{ui} - \bar{r}_i)(r_{uj} - \bar{r}_j)}{\sqrt{\sum_{u \in U_i \cap U_j} (r_{ui} - \bar{r}_i)^2 \cdot \sum_{u \in U_i \cap U_j} (r_{uj} - \bar{r}_j)^2}}$$

Cosine similarity

$$s_{ij} = \frac{\sum_{u \in U_i \cap U_j} r_{ui} \cdot r_{uj}}{\sqrt{\sum_{u \in U_i \cap U_j} r_{ui}^2 \cdot \sum_{u \in U_i \cap U_j} r_{uj}^2}}$$



# How to use similarity to recommend?

Given similarity measure between items  $s_{ij}$

Find set  $s_k(i, u)$  of  $k$ -nearest neighbors to movie  $i$  that were rated by user  $u$

Estimate rating using weighted average over the set of neighbors

$$\hat{r}_{ui} = \frac{\sum_{j \in s_k(i, u)} s_{ij} r_{uj}}{\sum_{j \in s_k(i, u)} s_{ij}}$$

# Normalization and bias problem

Problems:

- ▶ Some items are significantly higher rated
- ▶ Some users rate substantially lower
- ▶ Ratings change over time

Bias correction is crucial for collaborative filtering approaches

- ▶ global bias  $\mu$
- ▶ offset per user  $b_u$
- ▶ offset per movie  $b_i$
- ▶ time effects (ignore for now)

Baseline rating for (user, movie) is  $b(u, i) = \mu + b_u + b_i$

# Normalization and bias problem



Mean rating of all movies  $\mu = 3.7$

Troll hunter is 0.7 above mean ( $b_i = 0.7$ )

User rates 0.2 below mean ( $b_u = -0.2$ )

For this (user, movie) baseline rating is 4.2 stars

## Estimating biases

$$\min_b \sum_{(u,i):r(u,i)\neq?} (r(u,i) - \mu - b_u - b_i)^2 + \lambda(\sum_u b_u^2 + \sum_i b_i^2)$$

This is a linear model. Why?

## How to recommend with biases?

Similar to the approach earlier

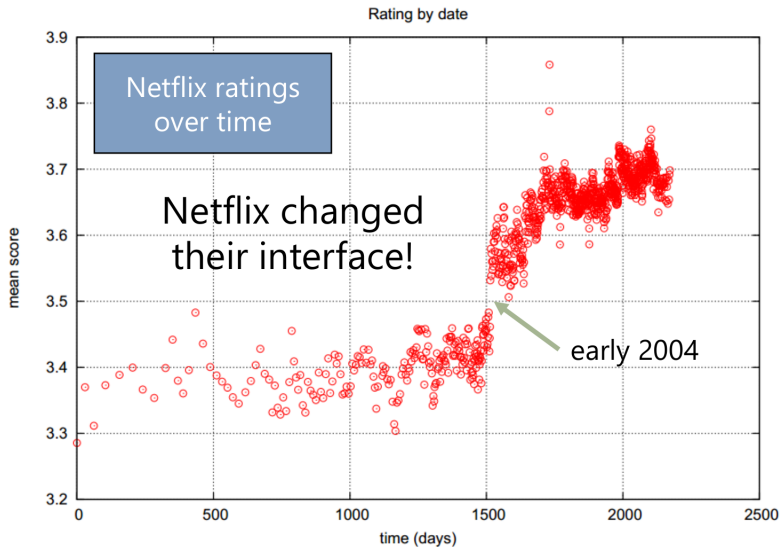
Given similarity measure between items  $s_{ij}$

Find set  $s_k(i, u)$  of  $k$ -nearest neighbors to movie  $i$  that were rated by user  $u$

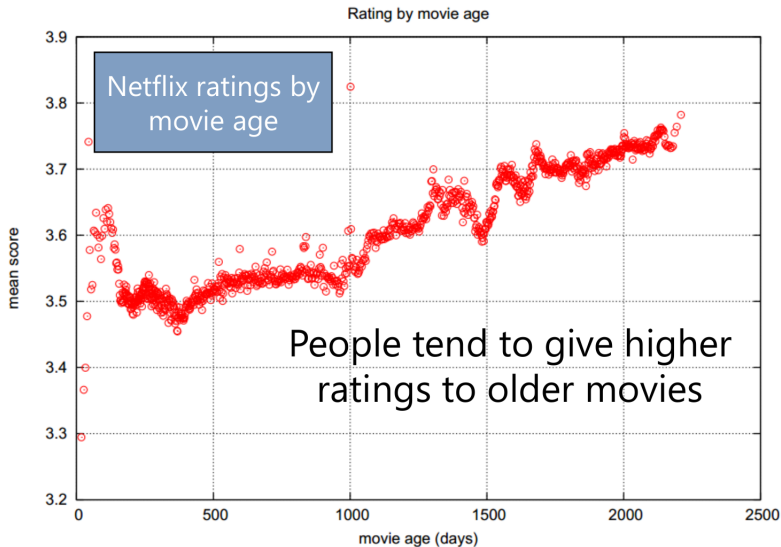
Estimate rating using weighted average over the set of neighbors

$$\hat{r}_{ui} = b_{ui} + \frac{\sum_{j \in s_k(i, u)} s_{ij} (r_{uj} - b_{uj})}{\sum_{j \in s_k(i, u)} s_{ij}}$$

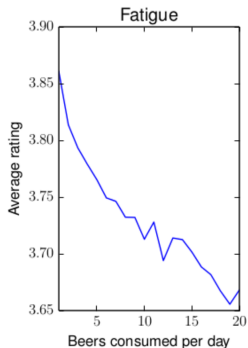
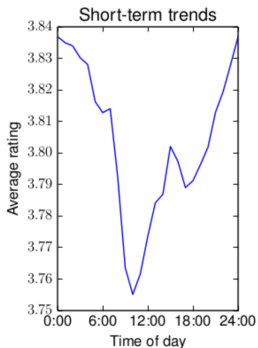
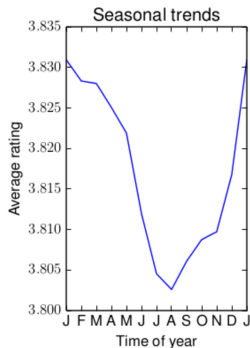
# Temporal effects



# Temporal effects



# Temporal effects



A few temporal effects from beer reviews



## Recommendations based on similarities

- ▶ intuitive
- ▶ there is no training
- ▶ easy to explain to a user
- ▶ widely used in practice
- ▶ surprisingly, collaborative filtering is extremely useful even though we have not looked at any features
- ▶ accuracy and scalability questionable
- ▶ cold start problem
- ▶ will not necessarily encourage diverse results

# Cold start problem

What happens with new users where we have no ratings yet?

- ▶ Recommend popular items.
- ▶ Have some start-up questions (for example, “tell me 10 movies you love”).

What do we do with new items?

- ▶ Content-based filtering techniques (that is, use features).
- ▶ Pay a focus group to rate them.

# Modelling approach to recommendations

So far we've looked at approaches that try to define some definition of user/user and item/item similarity.

Recommendation consists of

1. Finding an item  $i$  that a user likes (gives a high rating)
2. Recommending items that are similar to it (that is, items  $j$  with a similar rating profile to  $i$ )

Hinges on finding a good measure of similarity

# Modelling approach to recommendations

What we want to do next is to model the rating.

$$r_{ui} = f(\text{user}, \text{item}) + \text{noise}$$

Recommendation consists of identifying items with largest rating

$$\text{recommendation}(u) = \arg \max_{i \in \text{unseen items}} f(u, i)$$

# Netflix yardstick

## Netflix prize

- ▶ In 2006, Netflix created a dataset of 100,000,000 movie ratings
- ▶ Data looked like: (userID, itemID, time, rating)
- ▶ Whoever first manages to reduce the (R)MSE by 10% versus Netflix's solution wins \$1,000,000
- ▶ Data were de-anonymized — lawsuit against Netflix

## Root mean squared error for predicting ratings

$$\text{RMSE}(f) = \sqrt{\frac{1}{N} \sum_{u,i} (f(u, i) - r_{ui})^2}$$

Predicted rating  $\hat{r}_{ui} = f(u, i)$

# Netflix yardstick

A lot of research on minimizing the mean squared error.

Not clear that improving this metric will lead to better user experience.

When building a model we focus on minimizing root mean squared error.

## Latent factor model

Suppose we had  $K$  features of movies and users

Describe movie  $i$  with features  $q_i$

- ▶ How much is it action, romance, drama, ...

$$q_i = (0.9, 0.2, 0.5, \dots)$$

Describe user  $u$  with features  $p_u$

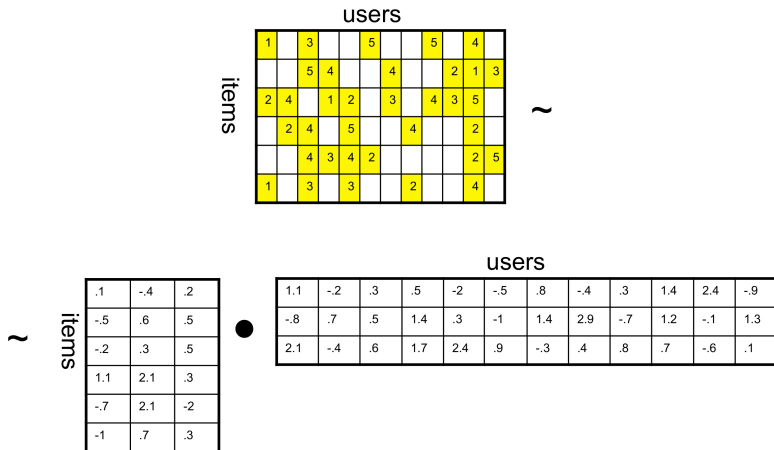
- ▶ How much she likes action, romance, drama, ...

$$p_u = (0.01, 0, 0.9, \dots)$$

$f(u, i)$  is the product of the two vectors

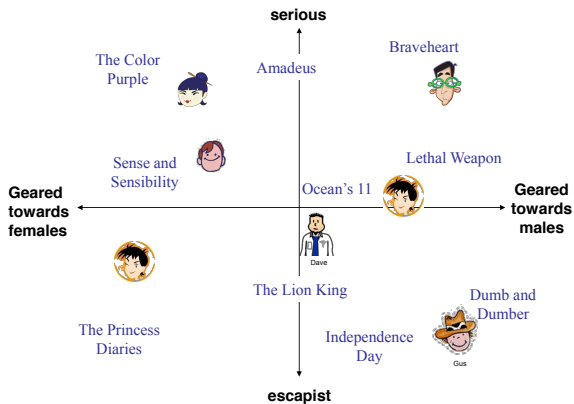
$$f(u, i) = 0.9 \cdot 0.01 + 0.2 \cdot 0 + 0.5 \cdot 0.9 + \dots$$

# Discovering features via matrix factorization





# Discovering features via matrix factorization



# Discovering features via matrix factorization

users

1	3			5		5		4		
		5	?	4			2	1	3	
2	4		1	2		3		4	3	5
	2	4		5			4			2
		4	3	4	2				2	5
1		3		3		2				4

items

~

~

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-.2
-1	.7	.3

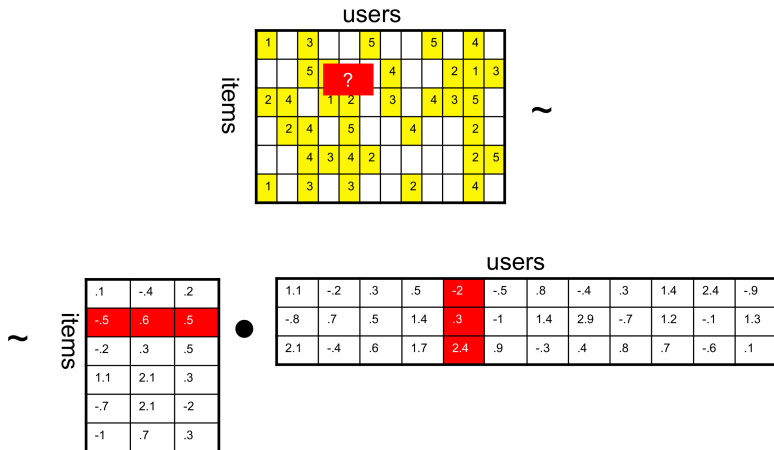
items

•

1.1	-.2	.3	.5	-.2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

users

# Discovering features via matrix factorization



# Discovering features via matrix factorization

users

1	3		5		5		4		
		5	<b>2.4</b>	4		2	1	3	
2	4		1	2	3	4	3	5	
	2	4		5		4		2	
		4	3	4	2			2	5
1		3		3		2			4

~

~

.1	-.4	.2
<b>-.5</b>	<b>.6</b>	<b>.5</b>
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

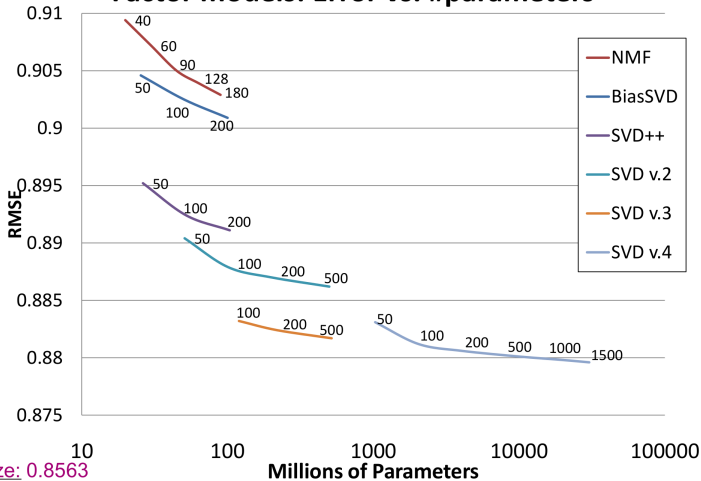


users

1.1	-.2	.3	.5	<b>-2</b>	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	<b>.3</b>	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	<b>2.4</b>	.9	-.3	.4	.8	.7	-.6	.1

Netflix: 0.9514

## Factor models: Error vs. #parameters



Prize: 0.8563

# Finding parameters of the latent factor model

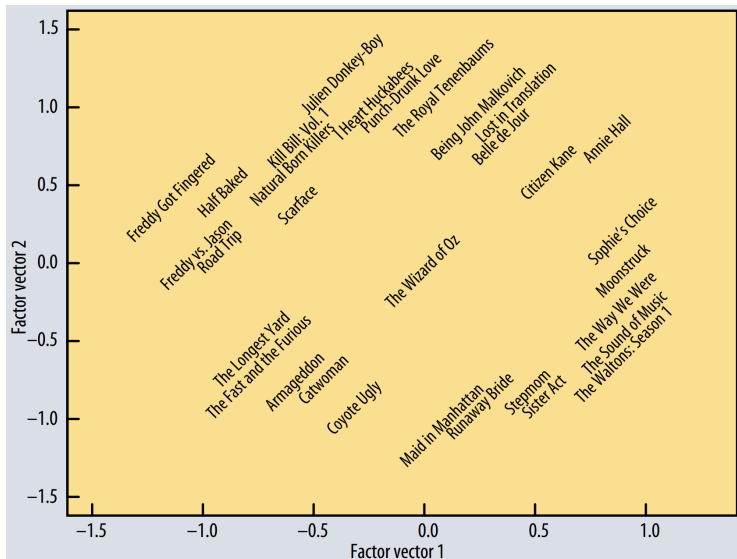
Model  $f(u, i) = p_u \cdot q_i$

We will find  $p$  and  $q$  by minimizing the following objective

$$\min_{p, q} \sum_{(u, i): r_{ui} \neq ?} (r_{ui} - p_u \cdot q_i)^2 + \lambda \left( \sum_{u, k} p_{u, k}^2 + \sum_{i, k} q_{i, k}^2 \right)$$

Stochastic gradient descent

# Visualizing the first two factors



# Visualizing the first two factors

First factor:

- ▶ left: lowbrow comedies and horror movies, aimed at a male or adolescent audience
- ▶ right: drama or comedy with serious undertones and strong female leads

Second factor:

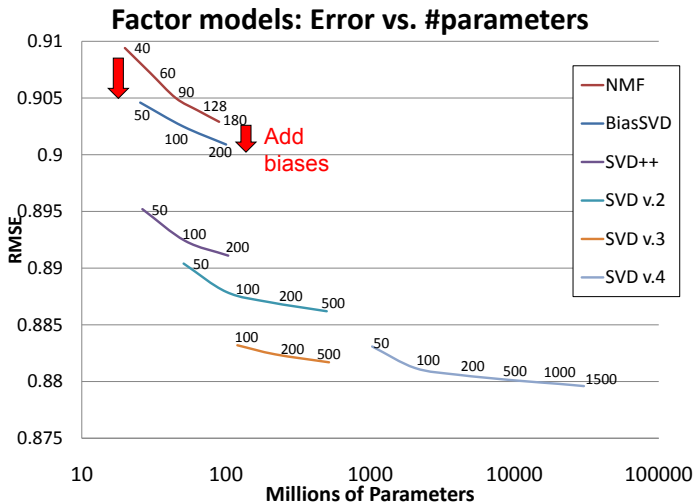
- ▶ top: independent, critically acclaimed, quirky films
- ▶ bottom: mainstream formulaic films



# Improvements

- ▶ adding bias terms
- ▶ adding features and implicit feedback
- ▶ modelling temporal effects

# Adding biases



$$f(u, i) = \mu + b_u + b_i + p_u \cdot q_i$$

# Combining real and discovered features

Real features capture context

- ▶ Time of the day, what I just saw, user info, what I bought in the past

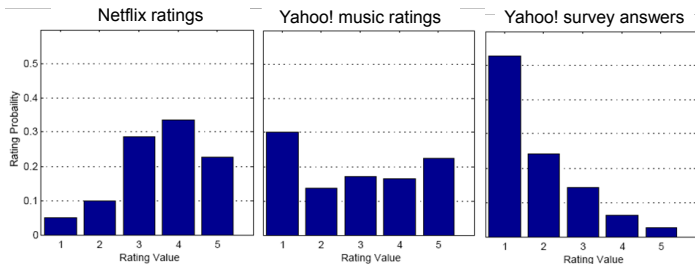
Discovered features from matrix factorization capture groups of users who behave similarly

- ▶ Users who like action movies and comedies

Mitigates cold-start problem

- ▶ Ratings for a new user from real features only
- ▶ As more information about user is discovered, matrix factorization “features” become more relevant

# Implicit information



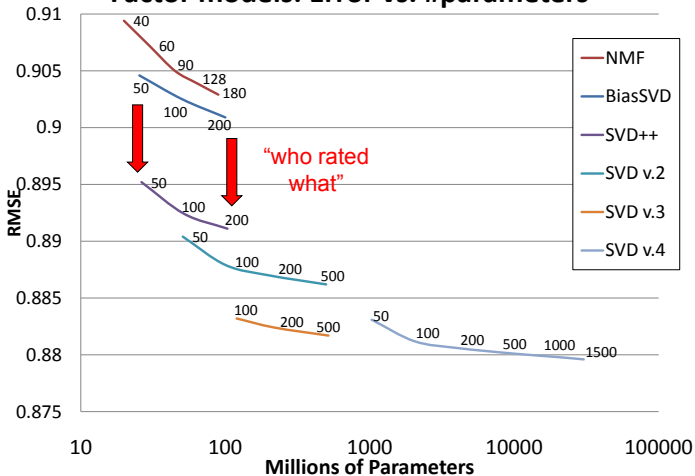
Our decision about whether to purchase a movie is a function of how we expect to rate it.

For items we have purchased, our decision to enter a rating or write a review is a function of our rating.

http:

[//www.cs.toronto.edu/~marlin/research/papers/cfmar-uai2007.pdf](http://www.cs.toronto.edu/~marlin/research/papers/cfmar-uai2007.pdf)

## Factor models: Error vs. #parameters



# Modeling temporal change

Time-dependent bias

Time-dependent user preferences

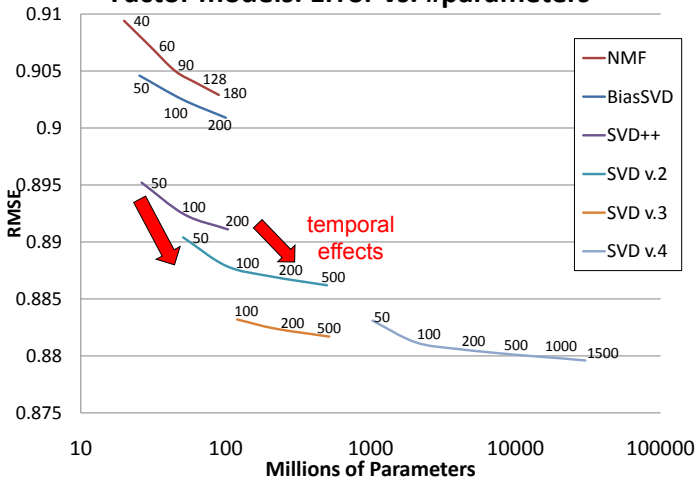
Parameterize functions  $b$  and  $p$

$$f(u, i, t) = \mu + b_u(t) + b_i(t) + q_i \cdot p_u(t)$$

Good parametrization is the key.

<http://www.cc.gatech.edu/~zha/CSE8801/CF/kdd-fp074-koren.pdf>

## Factor models: Error vs. #parameters



## Moral of the story

Increasing the number of parameters does not help much, but increasing the model complexity does.

\$1,000,000 seems to be incredibly cheap to get the amount of research that was devoted to the task.

The winning solution never made it into production at Netflix.

It is not clear that a solution which changes RMSE slightly will result in hugely improved user experience.

